

Assessment of Uncertainty on Estimation of Peak Flood Discharge Using LN2 Distribution for River Tapi at Sarangkhedha

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Abstract

Estimation of peak flood discharge (PFD) for a given return period is required for the design of culverts, dams, spillways, bridges, flood protection and soil conservation work, etc. The dimension and capacity of the hydraulic structures will also depend on design flood magnitude that may have some uncertainty due to model error. This paper presented a study assessment of uncertainty on estimation of PFD using LN2 distribution for river Tapi at Sarangkhedha site. For this purpose, the annual maximum discharge (AMD) series with different data length (say, series with 50 years data (DS1), series with 70 years data (DS2) and series with 82 years data (DS3) was generated from the observed AMD data (1941 to 2022) of Sarangkhedha and used for estimation of PFD. Goodness-of-Fit (GoF) (viz., Chi-Square and Kolmogorov–Smirnov tests and model performance indicators (viz., correlation coefficient (CC) and mean absolute error (MAE)) was applied for checking the adequacy of fitting three parameter estimation methods viz., method of moments (MoM), maximum likelihood method (MLM) and method of L-Moments (LMO) of LN2 to the series of AMD data. The GoF tests results supported the use of all three methods of LN2 for estimation of PFD for different return periods. The outcomes of the study indicated that (i) there is a good correlation between the observed and estimated AMDs by three methods of LN2, and the CC values vary from 0.985 to 0.991; (ii) the quantum of uncertainty in the estimated PFD measured through MAE by MoM, MLM and LMO is in decreasing order when the data length increases; and (iii) the MAE computed by MLM is minimum than those values of MoM and LMO while applying the DS1, DS2 and DS3 series for estimation of PFD. The study showed that the PFD given by MLM of LN2 distribution can be used for the design of civil and hydraulic structures.

Keywords: Chi-square, correlation coefficient, Kolmogorov–Smirnov, log normal, maximum likelihood method, mean absolute error, peak flood discharge

INTRODUCTION

Information on flood magnitudes and their frequencies is needed for the design of hydraulic structures, such as barrages, dams, spillways, culverts, road and railway bridges, urban drainage systems and flood plan zoning [1]. For this purpose, the flood frequency analysis (FFA) and unit hydrograph approach are widely applied despite development of various advanced accurate methods for computation of peak flood discharge (PFD) of a catchment. However, in this paper, the PFD was estimated through FFA by using the annual maximum discharge (AMD) series with different data length. In hydrological studies, the data and sampling errors and modelling or structural errors will generally influence the quantum of uncertainty in the estimated extreme values [2-5]. Also, the FFA with limited quantity

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of stream flow data may introduce sampling uncertainty. Hailegeorgis et al. [6] stated that the regional frequency analysis of extreme events is subject to the uncertainties of different sources that include:

- i. Data sampling related to the time period (viz., annual maximum or partial duration series), length of data series and quality of data;
- ii. Selection of frequency distribution to describe the data;
- iii. Parameter estimation; and
- iv. Regionalization and quantile estimation. Regarding this, the choice of probability distribution with suitable parameter estimation method is of immense importance to derive at a design flood for the desired frequency at a particular site.

Several probability distributions include extreme value type-1 (EV1), generalized extreme value (GEV), generalized pareto (GPO), 2-parameter log normal (LN2) and log Pearson type-3 (LP3) are generally available for FFA. Amin et al. [7] modelled the annual maximum rainfall in the northern regions of Pakistan by applying normal, LN2, LP3 and EV1 while Kamal et al. [8] applied the EV1 and LN2 for estimation of flood at Haridwar and Garhmukteshwar. Rosmaini and Saphira [9] estimated the monthly rainfall of Tuntungan, Tanjung Selamat, and Medan Selayang Stations in Medan City, Indonesia by adopting normal, Gamma and LN2. Study by Vivekanandan [10] revealed that the LP3 is best fit distribution for estimation of peak flood at Haora site. Kaur et al. [11] predicted the annual 1-day, 2-day and 3-day maximum rainfall values of Roorkee through EV1, LP3, LN2 and Ven Te Chow. Khan et al. [12] assessed the uncertainties in the estimated the peak flood by adopting EV1, GEV, GPO, LN2 and LP3. However, in this paper, the parameters of LN2 distribution were determined by method of moments (MoM), maximum likelihood method (MLM) and method of L-moments (LMO), and used for estimation of PFD for different return periods. The adequacy of fitting MoM, MLM and LMO of LN2 to the AMD series with different data length was examined through Goodness-of-Fit (GoF) tests (viz., Chi-Square (χ^2) and Kolmogorov–Smirnov(*KS*)) and model performance indicators (MPIs) using correlation coefficient (*CC*) and mean absolute error (*MAE*). This paper described the methodology adopted in assessing the uncertainty on estimation of PFD using LN2 distribution with an illustrative example and the results obtained thereof.

METHODOLOGY

Log Normal Distribution

The probability distribution function [$f(q)$] and cumulative distribution function [$F(q)$] of 2-parameter LN2 distribution [13] is expressed by:

$$f(q; \alpha, \beta) = \frac{1}{\beta \sqrt{2\pi}} e^{-\frac{(\ln(q)-\alpha)^2}{2\beta^2}} \quad \text{and} \quad F(q; \alpha, \beta) = \Phi\left(\frac{\ln(q)-\alpha}{\beta}\right) \quad 0 < q < \infty, \alpha > 0, \beta > 0 \quad (1)$$

where, q is the random variable [i.e., annual maximum discharge (AMD)], α is the scale parameter, β is the shape parameter, $F(q)$ is the cumulative distribution function (CDF) of q and $\Phi(\dots)$ is the CDF of the standard normal distribution. The parameters of LN2 were determined by MoM, MLM and LMO, and used to estimate the PFD [$q(T)$] for a given return period (T) through Equation (2), which is given as below:

$$q(T) = e^{\alpha + K(T)\beta} \quad (2)$$

where, $K(T)$ is the frequency factor for a return period (T) that can be derived from the following equations:

$$K(T) = W - \frac{2.515517 + 0.802853W + 0.00110328W^2}{1 + 1.43278W + 0.189269W^2 + 0.001308W^3} \quad (3)$$

$$W = \ln\left(\frac{1}{p^2}\right)^{0.5} \quad \text{with} \quad P = P[q(i)] = 1 - \frac{R[q(i)]}{N+1} \quad (4)$$

where, $P[q(i)]$ is the probability of exceedance of variable (q) of i^{th} sample, $R[q(i)]$ is the rank [R] assigned to each sample [$q(i)$] in such a way that $q(1) < q(2) < q(3) < \dots < q(N)$, W is the factor corresponding to P and N is the number of samples. The empirical equations involved in determining the parameters of LN2 by MoM, MLM and LMO are presented in Table 1.

Table 1. Determination of parameters of LN2 distribution by MoM, MLM and LMO.

Parameter	MoM	MLM	LMO
Scale (α)	$\alpha = -\frac{1}{2} \ln \left(\frac{\sum_{i=1}^N [q(i)]^2}{\sum_{i=1}^N q(i)} \right) + 2 \ln \left(\frac{\sum_{i=1}^N q(i)}{N} \right) - \frac{3}{2} \ln(N)$	$\alpha = \frac{1}{N} \sum_{i=1}^N \ln[q(i)]$	$\alpha = \lambda_1 = b_0 = \frac{1}{N} \sum_{i=1}^N \ln[q(i)]$
Shape (β)	$\beta = \left[\ln \left(\frac{\sum_{i=1}^N [q(i)]^2}{\sum_{i=1}^N q(i)} \right) - 2 \ln \left(\frac{\sum_{i=1}^N q(i)}{N} \right) + \ln(N) \right]^{1/2}$	$\beta = \left(\frac{1}{N} \sum_{i=1}^N (\ln[q(i)] - \alpha)^2 \right)^{1/2}$	$\lambda_2 = 2b_1 - b_0 = \beta / \sqrt{\pi}$ $b_1 = \frac{1}{N(N-1)} \sum_{i=2}^N (i-1) \ln[q(i)]$

In Table 1, $q(i)$ is the observed AMD for i^{th} sample, $\ln[q(i)]$ is the logarithmic value of $q(i)$, λ_1 and λ_2 are the first and second LMOs, and b_0 and b_1 are the first and second moments of the sample [14].

Computation of Standard Error

The standard error (SE) in the estimated PFD by three methods of LN2 was computed from Equations (5 and 6), which is given as below:

$$SE[q(T)] = \left(\frac{1}{2} \right) \left[q(T) \left(e^{SE[y(T)]} - e^{-SE[y(T)]} \right) \right] \quad (5)$$

$$SE[y(T)] = \left(\frac{\beta}{\sqrt{N}} \right) \left(1 + 0.5K(T)^2 \right)^{0.5} \quad (6)$$

The lower and upper confidence limits (LCL and UCL) of the estimated PFD for a given return period (T) at 95% level were computed from $LCL = q(T) - 1.96 * SE[q(T)]$ and $UCL = q(T) + 1.96 * SE[q(T)]$

Goodness-of-Fit Tests

Theoretical descriptions of GoF tests [15] viz., Chi-square (χ^2) and Kolmogorov–Smirnov (KS) applied in checking the adequacy of fitting MoM, MLM and LMO of LN2 to the AMD series with different data length are given as below:

$$\chi^2 = \sum_{j=1}^{NC} \frac{[O_j(q) - E_j(q)]^2}{E_j(q)} \quad (7)$$

$$KS = \text{Max} \sum_{i=1}^N |F_e[q(i)] - F_D[q(i)]| \quad (8)$$

where, $O_j(q)$ and $E_j(q)$ are the observed and expected frequency values of q in j^{th} class, NC is the number of frequency class, $F_D[q(i)]$ is the derived CDF of $q(i)$ by LN2 and $F_e[q(i)]$ is the empirical CDF of $q(i)$ using Weibull plotting position formula for $I = 1, 2, 3, \dots, N$ with $q(1) < q(2) < \dots < q(N)$. If the computed values of GoF tests statistic given by the method are not greater than its theoretical value at 5% significance level then the selected method is acceptable for estimation of PFD.

Model Performance Analysis

The performance of the MoM, MLM and LMO estimators of LN2 applied in estimation of PFD was evaluated through model performance indicators (MPIs) viz., correlation coefficient (CC) and mean absolute error (MAE) [16], which are described by:

$$CC = \frac{\sum_{i=1}^N [q(i) - \mu(q)][z(i) - \mu(z)]}{\sqrt{\sum_{i=1}^N [q(i) - \mu(q)]^2 \sum_{i=1}^N [z(i) - \mu(z)]^2}} \quad (9)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |q(i) - z(i)| \quad (10)$$

where, $\mu(q)$ is the average of the observed AMDs, $z(i)$ is the estimated AMD of i^{th} sample and $\mu(z)$ is the average of the estimated AMDs. The method with high CC (say, $CC > 0.9$) and minimum MAE was adjudged as better suited method for estimation of PFD.

APPLICATION

In this paper, a study on assessment of uncertainty on estimation of PFD using MoM, MLM and LMO of LN2 distribution for river Tapi at Sarangkhedha was carried out. The Prakasha barrage medium irrigation project is constructed in the Tapi river basin in Nandurbar district of Maharashtra. The Prakasha barrage is located between the latitude $21^{\circ} 30' 44''$ N and longitude $74^{\circ} 20' 42''$ E and at about 125 km from Ukai dam. The water releases from Hathnur dam located in the upper catchment area of river Tapi crosses three barrages at Sulwade, Sarangkhedha and Prakasha before reaching Ukai. The catchment area of Ukai reservoir is located at 100 km from Surat city and lies in the upper and lower basins of river Tapi in Madhya Pradesh and Maharashtra. For the present study, the AMD series was derived from the daily stream flow data (1941 to 2022) observed at Sarangkhedha site and used for estimation of PFD for different return periods for the design purposes of Prakasha barrage.

RESULTS AND DISCUSSION

By using the observed AMD data, three series with different data length, say DS1 (series with 50 years data), DS2 (series with 70 years data) and DS3 (series with 82 years data) was generated and used for the assessment of uncertainty in the estimated PFD by adopting three methods of LN2. The results obtained from the study are presented in the following sections.

Estimation of PFD Using LN2

The descriptive statistics of the observed AMDs of DS1, DS2 and DS3 series applied in FFA is presented in Table 2. From the descriptive statistics, it was observed that the average of the AMDs for the DS1 (1941 to 1990) series is higher than those values of DS2 (1941 to 2010) and DS3 (1941 to 2022). The percentages of coefficient of variance computed by the AMDs pertaining to the DS1, DS2 and DS3 series vary between 72.3 and 74.9. Also, from the descriptive statistics of the observed AMDs, the higher order moments (C_s and C_k) pertaining to the DS3 series are less than those values of DS1 and DS2. The estimated PFD for different return periods by MoM, MLM and LMO of LN2 from DS1, DS2 and DS3 series are presented in Tables 3 to 5 whereas the plots are shown in Figure 1(a-c).

Table 2. Descriptive statistics of the observed AMDs of DS1, DS2 and DS3 series.

Descriptive Statistics	DS1	DS2	DS3
Average (in cumecs)	12875.5	12352.9	12458.2
Standard deviation (in cumecs)	9494.9	9247.1	9001.9
Coefficient of skewness (C_s)	1.315	1.397	1.301
Coefficient of kurtosis (C_k)	1.606	1.571	1.352

Table 3. Estimated PFD [$q(T)$ in cumecs] with standard error (SE) from DS1 series by LN2.

Return period (year)	MoM		MLM		LMO	
	$q(T)$	SE	$q(T)$	SE	$q(T)$	SE
2	9691.9	1196.1	9692.0	1184.0	9692.0	1062.9
5	19336.2	2586.0	19202.5	2542.1	17911.0	2128.5
10	27743.8	3828.7	27452.2	3750.2	24690.7	3027.7
20	37380.8	5243.4	36877.3	5120.4	32185.7	4011.5
25	40772.8	5738.1	40188.5	5598.7	34769.9	4347.9
50	52285.8	7407.4	51407.9	7209.3	43375.3	5460.0
75	58473.9	8302.5	57427.5	8071.5	47911.1	6044.4
100	65394.4	9295.5	64152.0	9026.6	52921.1	6683.9

Table 4. Estimated PFD [$q(T)$ in cumecs] with standard error (SE) from DS2 series by LN2.

Return period (year)	MoM		MLM		LMO	
	$q(T)$	SE	$q(T)$	SE	$q(T)$	SE
2	9454.6	921.9	9454.7	915.3	9454.7	858.9
5	18044.4	1906.4	17961.1	1884.0	17266.9	1709.6
10	25297.0	2757.7	25119.2	2718.7	23656.0	2405.5
20	33437.7	3704.9	33136.5	3645.1	30679.7	3166.8
25	36269.0	4031.9	35921.4	3964.5	33093.7	3427.3
50	45773.4	5122.3	45259.0	5028.3	41108.7	4285.6
75	50824.3	5700.2	50215.6	5591.4	45320.4	4735.2
100	56432.7	6336.2	55714.9	6210.6	49963.7	5226.1

Table 5. Estimated PFD [$q(T)$ in cumecs] with standard error (SE) from DS3 series by LN2.

Return period (year)	MoM		MLM		LMO	
	$q(T)$	SE	$q(T)$	SE	$q(T)$	SE
2	9652.4	919.5	9652.4	913.8	9652.4	867.9
5	18149.9	1873.3	18079.8	1854.6	17519.5	1706.7
10	25247.8	2688.8	25099.6	2656.6	23924.7	2404.8
20	33159.2	3589.2	32909.6	3540.3	30945.7	3161.4
25	35899.7	3898.7	35612.2	3843.7	33354.7	3418.8
50	45065.3	4926.6	44642.1	4850.3	41340.9	4265.5
75	49917.8	5469.2	49418.1	5381.1	45530.9	4708.2
100	55292.8	6064.8	54705.0	5963.4	50145.6	5191.2

From Tables 3 to 5 and Figure 1(a-c), it was found that (i) the estimated PFD by MoM is higher those values of MLM and LMO for the return periods from 5-year to 100-year; (ii) the SE s in the estimated PFDs are in decreasing order when data length increases; and (iii) the SE s in the estimated PFDs by LMO through DS1, DS2 and DS3 series are minimum than those values of MoM and MLM.

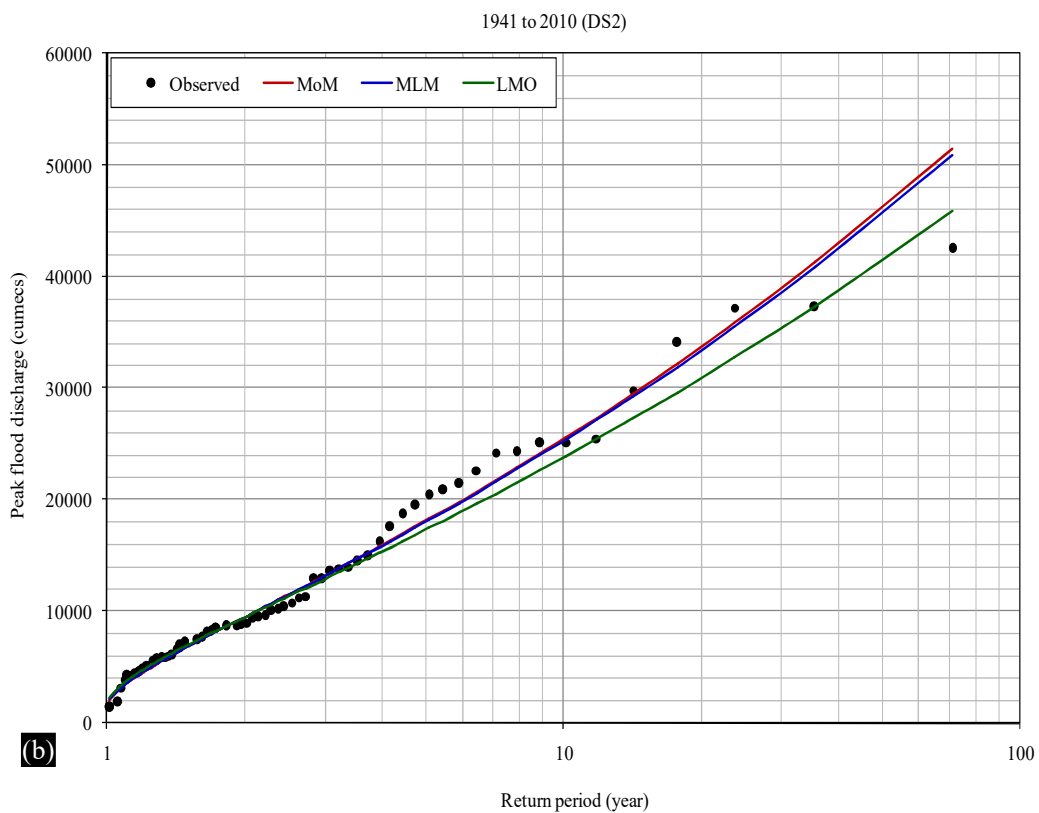
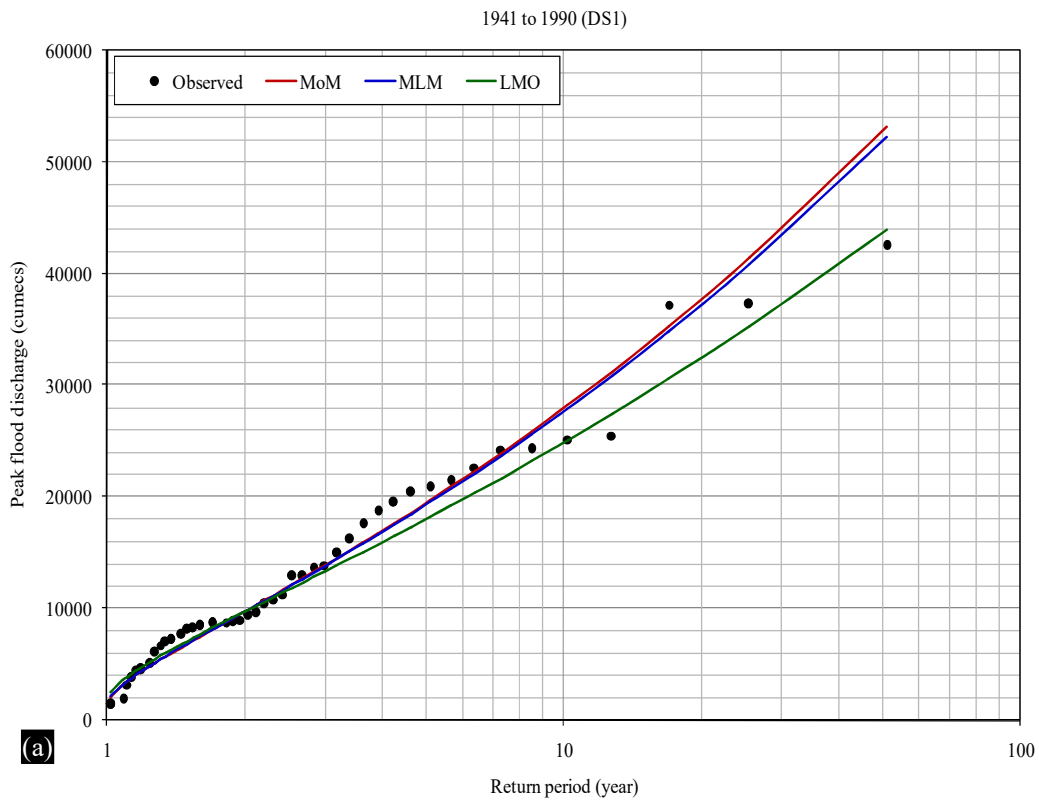
Evaluation of Results by GoF Tests

The GoF (viz., χ^2 and KS) tests were applied for checking the adequacy of fitting three methods (viz., MoM, MLM and LMO) of LN2 to the AMD series with different data length and are given in Table 6. The degree of freedom was considered as 4 for DS1 and 7 for DS2 and DS3 while computing the χ^2 statistic values.

Table 6. Theoretical and computed values of GoF tests by LN2.

Data series	χ^2				KS			
	Theoretical Value	Computed by			Theoretical Value	Computed by		
		MoM	MLM	LMO		MoM	MLM	LMO
DS1	9.488	8.800	8.940	9.080	0.188	0.076	0.077	0.078
DS2	14.067	6.571	7.143	7.714	0.163	0.065	0.063	0.060
DS3	14.067	7.024	7.512	8.000	0.150	0.079	0.076	0.073

From Table 6, it was witnessed that the computed values of the GoF tests statistic by MoM, MLM and LMO of LN2 are not greater than its theoretical values at 5% significance level, and at this level, these three methods are adequate for estimation of PFD.



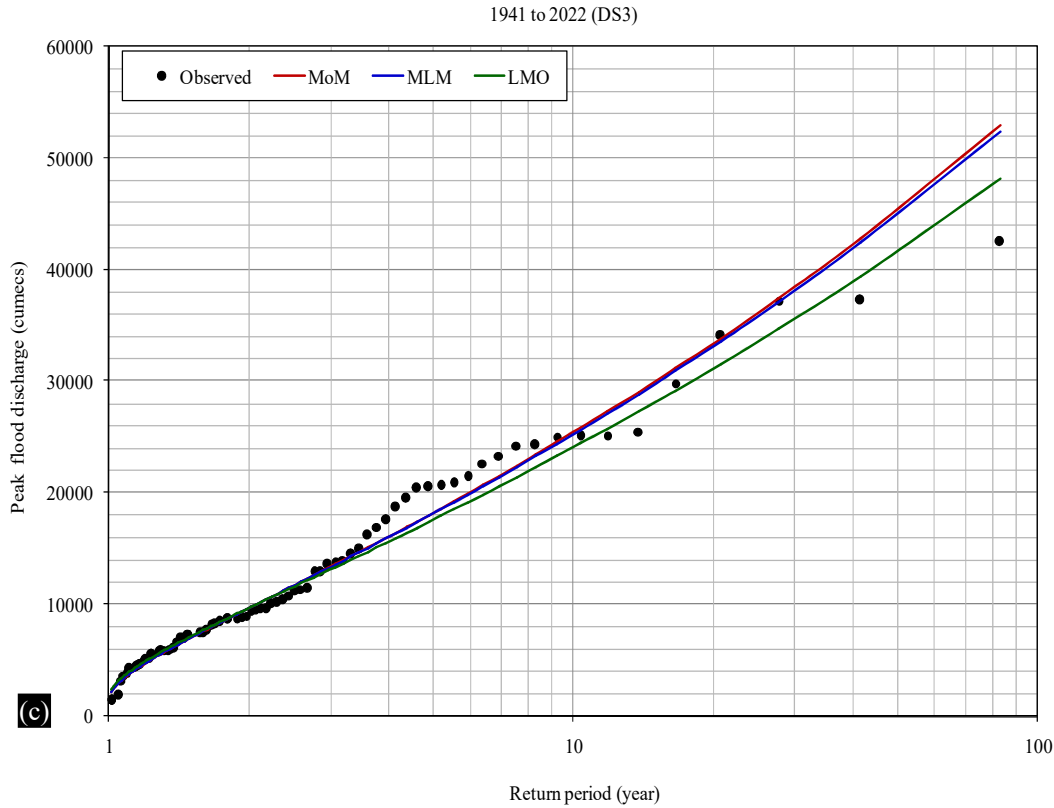


Figure 1. (a-c) Estimated PFD for different return periods by MoM, MLM and LMO of LN2 for DS1, DS2 and DS3.

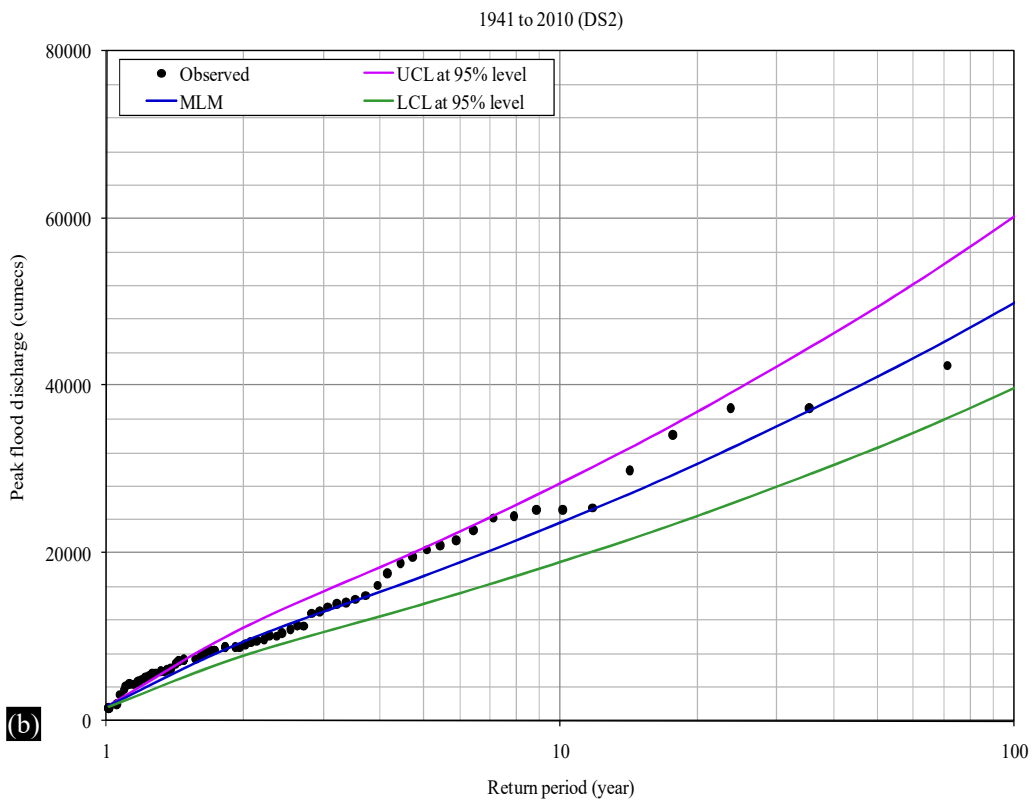
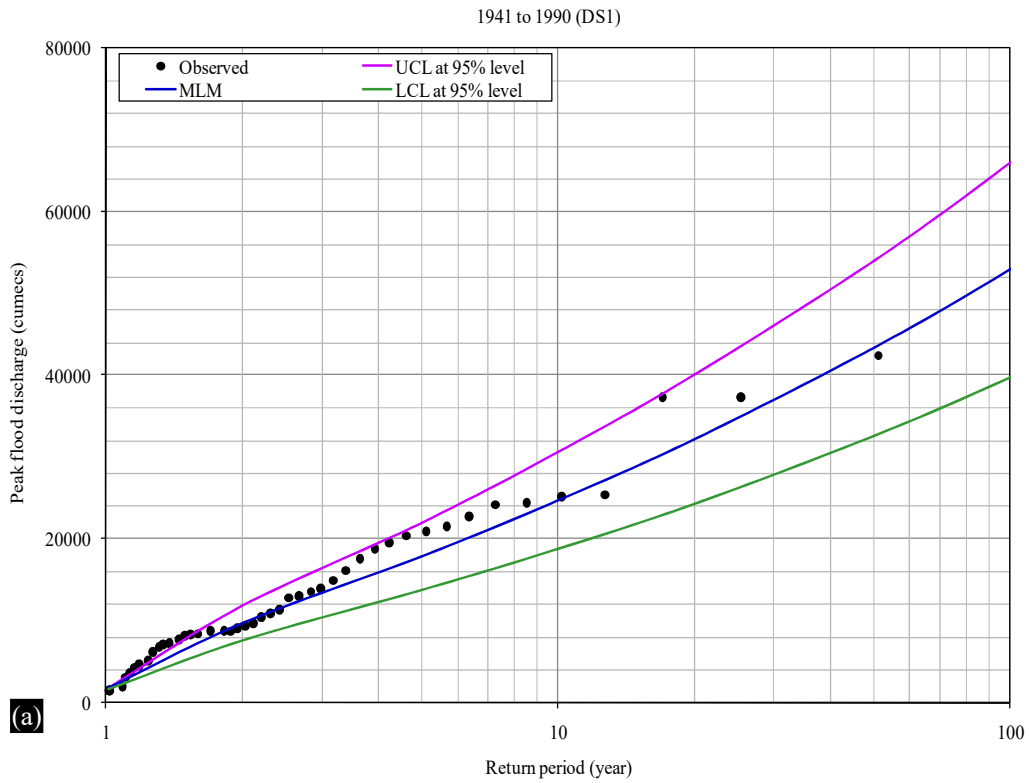
Evaluation of Results by MPIs

By using the AMD series with different data length (viz., DS1, DS2 and DS3), the estimated PFD by MoM, MLM and LMO of LN2 were evaluated through MPIs using *CC* and *MAE*; and the results are presented in Table 7. From the MPIs values, it was noticed that (i) there is a very good correlation between the observed and estimated PFDs by three methods of LN2, and the *CC* values vary from 0.985 to 0.991; (ii) the quantum of uncertainty in the estimated PFDs measured through *MAE* by MoM, MLM and LMO is in decreasing order when data length increases; and (iii) the *MAE* computed by MLM is minimum than those values of MoM and LMO applied in flood estimation.

Table 7. Computed values of MPIs by three methods of LN2.

Data series	CC			MAE (in cumecs)		
	MoM	MLM	LMO	MoM	MLM	LMO
DS1	0.985	0.985	0.989	1209.5	1192.2	1193.6
DS2	0.988	0.989	0.991	873.3	872.4	962.7
DS3	0.986	0.986	0.988	556.0	555.0	651.5

Based on the analysis of results through GoF and diagnostic tests, it was identified that the MLM of LN2 is better suited while using the DS1, DS2 and DS3 series for estimation of PFD for River Tapi at Sarangkherda. Figure 2(a-c) presents the estimated PFD by MLM with 95% confidence limits together with the observed AMD for DS1, DS2 and DS3 wherein it can be witnessed that the percentages of the observed AMD covered by the fitted lines of the estimated PFD by MLM of LN2 are about 14% for DS1, 10% for DS2 and 9% for DS3.



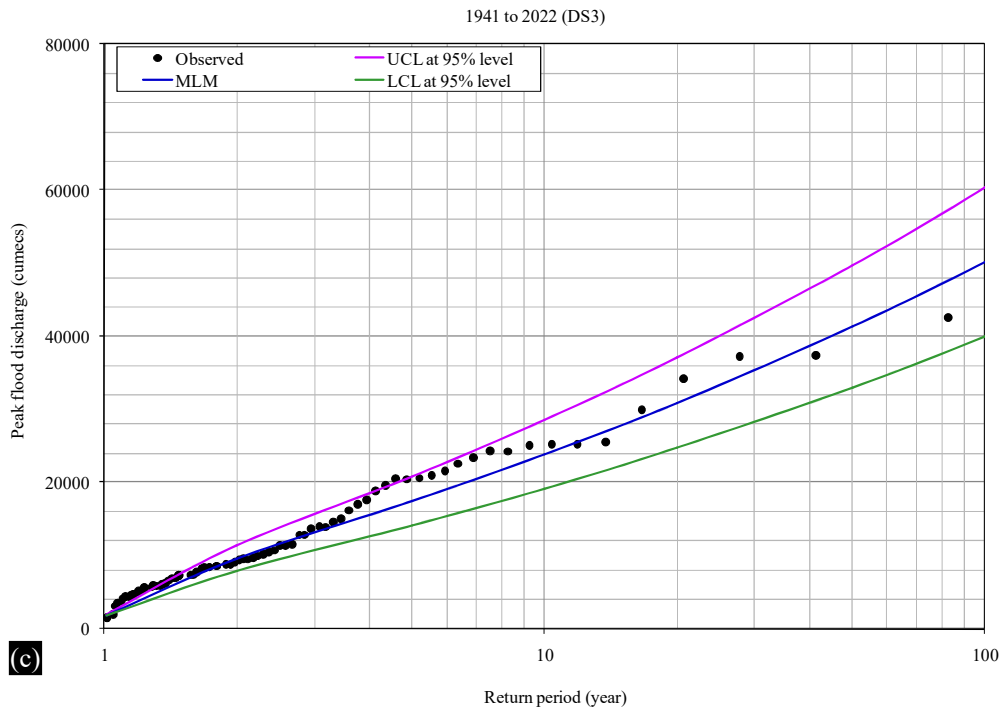


Figure 2. (a-c) Estimated PFD by MLM of LN2 with 95% confidence limits and observed AMDs of DS1, DS2 and DS3.

CONCLUSION

This paper presented a study on assessment of uncertainty on estimation of PFD using MoM, MLM and LMO of LN2 distribution for river Tapi at Sarangkhedha. For this purpose, three AMD series with different data length (say, DS1, DS2 and DS3) was generated from the observed AMD data of Sarangkhedha was used. The GoF (viz., χ^2 and KS) tests and diagnostic test through MPIs (viz., CC and MAE) was applied for identifying a best fit method of LN2 for estimation of PFD. Based on the results of the data analysis, the conclusions drawn from the study were summarized and are presented below:

- GoF tests results supported the use of MoM, MLM and LMO of LN2 for estimation of PFD for different return periods.
- The performance of three methods of LN2 applied in flood estimation was evaluated by CC and the values vary between 0.985 and 0.991.
- The quantum of uncertainty in the estimated PFDs measured through MAE by MoM, MLM and LMO is in decreasing order when data length increases.
- The MAE in the estimated PFD computed by MLM is found as minimum than those values of MoM and LMO applied in flood estimation.
- Based on GoF tests and MPIs, it was found that MLM is better suited while applying the DS1, DS2 and DS3 series for estimation of PFD for different return periods.

The study suggested that the estimated PFDs for different return periods at Sarangkhedha site, as given in Tables 3 to 5, could be considered for the planning, design and management of civil and hydraulic structures in river Tapi, and for the design purpose of Prakasha barrage.

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