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Investigation of Tangential Shear Stress and Frictional Torque Coefficient over a Rotating Disk of the Rotor-Stator System

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Abstract

The study investigates the prediction of tangential shear stress and the frictional torque coefficient over a rotating disk in a rotor-stator system under laminar inward flow conditions. The interaction between the rotating and stationary disks is governed by key parameters, such as volumetric flow rate, rotational speed of the disk, and the axial clearance between the two surfaces. These parameters are expressed in terms of dimensionless quantities, including the throughflow Reynolds number, gap ratio, and rotational Reynolds number, to generalize the findings. To analyze the influence of these parameters, an analytical model is developed by simplifying the Navier-Stokes equations. The gap ratio between the rotating and stationary disk is systematically varied from 0.0125 to 0.05 to understand its effect on flow characteristics. The study considers three fixed throughflow Reynolds numbers of 50, 500, and 800 while varying the rotational Reynolds number in the range of 3000 to 10,000. The results indicate that dimensionless parameters have a substantial impact on the tangential shear stress distribution and frictional torque coefficient within the rotor-stator system. The findings provide critical insights into the fluid dynamics of rotating machinery, contributing to a better understanding of frictional losses and performance optimization. The study highlights the significance of axial clearance and rotational speed in determining shear stress and torque characteristics, which is valuable for applications involving enclosed rotor-stator flows, such as turbomachinery and industrial fluid handling systems.

Keywords: Rotor-stator system, gap ratio, rotational Reynolds number, throughflow Reynolds number, inward flow

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INTRODUCTION

There are two types of rotating disks: (a) a "free rotating disk," which rotates infinite fluid domain that initially at rest, and (b) an "enclosed rotating disk," which rotates within a finite fluid domain, meaning only a limited amount of fluid volume is affected by the disk's rotation. The paper focuses on the second type of rotating disk which is relevant to problems encountered in centrifugal machinery, such as power loss, heat transfer and disk friction and torque related flows induced by the rotating disk. These flows are influenced by the geometries of the rotating disk and its enclosure. The investigation aims to clarify how different parameters affect the frictional torque and shear stress of rotating component in rotor stator system.

The investigation of fluid flow between disks has been study by several researchers. McGinn [1]: An expression has been formulated to represent pressure distribution, while Garcia [2] conducted experiments on turbulent flow, and it was concluded that the flow exhibited inherently instability. Murphy et al. [3, 4]: Numerical and experimental investigations were performed, indicating that the flow could remain laminar up to a Reynolds number of 20,700. Lee and Lin [5] closed form of solutions was developed to forecast pressure distribution; they were discovered to match well with the experimental results. Soo [6] obtained a solution for the pressure and velocity fields by solving Navier's stroke equation for a rotor stator system. Dorfman [7] developed an equation for calculating torque coefficient (C_m) for rough surfaces. Daily and Nece [8] established correlations for the torque coefficients in laminar and turbulent flow regimes. Bayley and Owen [9] conducted experiments to investigate how various rotating speed of disks affect the flow within a cavity. They were able to obtain the torque coefficient and make theoretical predictions based on their findings. Dorfman [10]: The effect of surface roughness on the torque coefficient was examined in a disk system. Kurokawa et al. [11]: The impact of roughness on the flow within a rotor stator cavity was investigated, and analytical finding were obtained that agreed well with experimental data for both rough and smooth surfaces. Kurokawa et al. [12]: Investigated how changes in throughflow coefficient affecting the torque coefficient in a cavity containing a rotor and stator. Dibelius et al. [13] performed experiments on a shrouded rotor-stator with variable geometry that feature both inward and outward throughflow. Owen [14]: The Ekman layer equations were used to predict the core rotation in a radial outflow rotor stator system. He then compares the prediction values with experimental data and found that the computed data underestimated core rotation in laminar and turbulent regimes. However, the experimental values in laminar flow without superimposed flow were better agreement with calculated data to address this discrepancy he applied a solution for turbulent moment integral equations in which they considered the rotation of the core in rotor stator system, thereby, improving the fit between the calculated and experimental values. Schlichting and Gersten [15] formulated an implicit equation for the torque coefficient in turbulent flow by using the findings of Goldstein [16] as a basis. Singh A. [17] developed an analytical equation that describes the velocity and pressure field in laminar flow for stationary and rotatory disks system. Eunok et al. [18] employed direct numerical simulation to examine the boundary layer of a rotor stator system, increasing the rotation Reynolds number from 0 to 4×10^5 in their analysis. Through their research, they were able to observe the transition of laminar to turbulent flow regimes. Luo et al. [19] conducted tests on rotor stator cavity. They obtained pressure distribution data on the rotor at various rotating speed and flow rates and determine the torque coefficient. Gu et al. [20] presented a theoretical model to address the problem of leakage in centrifugal pumps. their approach involved developing a pressure model based on Poncet's K formula for a simplified rotor stator geometry. The results obtained using this model showed good accuracy compared to previous researcher by incorporating a conventional constant K.

ANALYTICAL MODEL

Figure 1 depicted in below comprises of a circular disk that is constantly rotating at an angular velocity of ω within an incompressible fluid. The disk is positioned a distance h away from a fixed stationary disk. The fluid flow symmetrically inward from the axis of rotation z, at a mass flow rate of m. The equations that govern the flow, related to the rotor stator system are expressed through the continuity and momentum equations in cylindrical coordinated. Below are the corresponding equations:

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = \frac{-1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] \tag{2}$$

$$\frac{u}{r}\frac{\partial rv}{\partial r} + w\frac{\partial v}{\partial z} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right] \tag{3}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \frac{-1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \frac{u}{r^2} + \frac{\partial^2 w}{\partial z^2}\right] \tag{4}$$

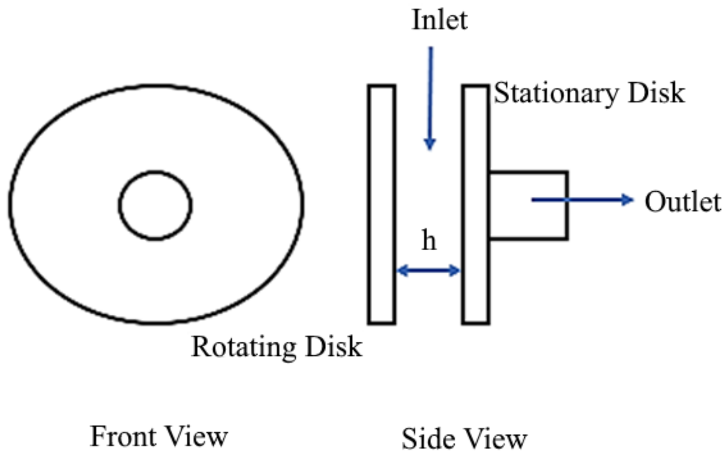

Figure 1. Coordination system of physical model.

This problem has been studied based on the following set of assumptions:

1. The variables' derivatives with respect to θ are zero, indicating that the flow exhibits axisymmetric properties.
2. The axial velocity component as compared to tangential and radial velocity components are considerably smaller, or in other words, it is considerably negligibly small.
3. The fluid flow radially towards the outer periphery in a non-swirling manner. Additionally, the overall within the gap between the disks are directed inward flow.

Equations (1) to (4) can be simplified by considering the sets of assumptions:

$$\frac{\partial u}{\partial r}+\frac{u}{r}=0 \tag{5}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\frac{1}{\rho}\frac{\partial \tau_r}{\partial z} \tag{6}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\frac{1}{\rho}\frac{\partial \tau_\phi}{\partial z} \tag{7}$$

$$\frac{-1}{\rho}\frac{\partial p}{\partial z}=0 \tag{8}$$

The boundary conditions are:

$$u(r,0)=0;\, u(r,z_0)=0;$$

$$v(r,0)=r\omega;\, v(r,z_0)=0;$$

$$w(r,0)=0;\, w(r,0)=0;$$

To solve for the tangential velocity component, Equation (9) is converted into ordinary differential equation using Karman's rule:

$$v=r\omega f(z) \tag{9}$$

By substituting the expression of v into Equation (7), we obtain the following Expression:

$$f''=\frac{-2u}{vr}f \tag{10}$$

Substituting u with the mean velocity in Equation (10) as proposed by lee and Lin [1985].

$$f''+m^2f=0 \tag{11}$$

where $m = \left(\frac{Q}{\pi r^2 h \nu}\right)^{\frac{1}{2}}$.

The boundary conditions for Equation (11) are as follows:

$f = 0 at z = 0$

$f = 1 at z = h$

The solutions of Equation (11) can be expressed as:

$$f = \frac{sin\ (mz)}{sin\ (mh)} \tag{12}$$

By combining Equations (9) and (12), we can determine the tangential velocity profile as follows:

$$v = r\omega \frac{sin(mz)}{sin(mh)} \tag{13}$$

Equation (13) is same as derived by Singh A. [2014], the tangential shear stress on rotating disk can be expressed as:

$$\tau_{\emptyset} = \mu \frac{\partial v}{\partial z} \tag{14}$$

And Figure 2 shows the Rotating disk.

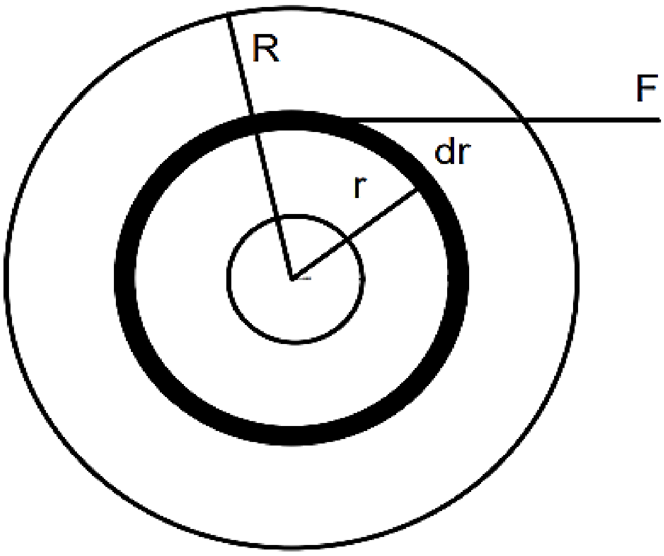

Figure 2. Rotating disk.

Together Equations (13) and (14) shear stress becomes

$$\tau_{\emptyset} = \mu\omega(2Req)^{\frac{1}{2}} cot\ \left(G(2Req)^{\frac{1}{2}}\right) \tag{15}$$

To determine the resisting torque caused by friction, the shearing unit stress over the surface area of the rotating disk must be integrated. The friction torque can be expressed as:

$$\int_0^R dM = \int_0^R \mu\omega(2Req)^{1/2} r^2\ cot(mh)dr \tag{16}$$

$$M = \frac{2}{3}\pi\mu\omega(2Req)^{1/2}cot\ \left(G(2Req)^{\frac{1}{2}}\right)R_2^3 \quad (17)$$

The torque coefficient is:

$$C_m = \frac{M}{\frac{1}{2}\rho\omega^2R^5} \quad (18)$$

In this context, "R_{eq}" refers to the throughflow Reynolds number, which is calculated as $\frac{Q}{(2\pi h v)}$, while "G" denotes the gap ratio, defined as $\frac{h}{R}$.

All the nomenclature shown in Table 1.

Table 1. Nomenclature.

r_1	Inner radius of stationary disk
R	Outer radius of rotating
G	Gap ratio, $\frac{h}{R}$
r	Radial coordinate
U	Average radial velocity, $\frac{Q}{(2\pi hR)}$
h	Distance between disks
µ	Dynamic viscosity
v	Tangential velocity
m	Mass flow rate
$\tau_\emptyset$	Tangential shear stress
ρ	Density
ω	Angular velocity
ν	Kinematic viscosity
R_{eq}	Throughflow Reynolds number, $\frac{Q}{(2\pi h\nu)}$
$R_{e\phi}$	Reynolds numbers, $\frac{\omega R^2}{\nu}$
z	Axial coordinate

RESULTS AND DISCUSSION

To study the impact of fluid's viscous dissipation on a rotating disk, we began by examining the tangential shear stress profile on the disk and subsequently evaluated the torque coefficient. The influence of the throughflow Reynolds numbers, rotational Reynolds numbers and gaps ratio was summarized. The results of analytical model with the Soo model are in good agreement. The initial step involves comparing the outcomes of Equation (17) with those obtained from Soo's analytical model. Figure 3 shows that the results for a throughflow Reynolds number of 22 at a fixed gap ratio of 0.0125 with different rotational Reynolds numbers are compared, as the results are closest.

Figures 4 to 7 show graphs with the bottom axis displaying the rotational Reynolds number and the vertical axis showing the tangential shear stress distribution and torque coefficient. Figure 4 illustrates the change in the distribution of shear stress on a rotating disk at various rotational Reynolds numbers for a fixed gap ratio of 0.0125, while three different throughflow Reynolds numbers are considered. Equation (15) shows that the shear stress is maximum at the outer radius of the disk. This occurs b cause at the outer radius of the rotating disk, the tangential velocity is maximum, whereas it is minimum at the centre of the disk. The tangential shear stress profile, plotted in Figure 4, illustrates the effect of three different throughflow Reynolds numbers (50, 500, and 800) on the tangential shear stress at varying rotational Reynolds numbers while keeping the gap ratio fixed at 0.0125. The results demonstrate that higher throughflow Reynolds numbers lead to an increase in tangential shear stress. This indicates that the throughflow Reynolds number has a significant effect on the shear stress profile. Furthermore, in Figure 5, for a fixed throughflow Reynolds number, with three gap ratios of 0.0125, 0.025, and 0.050 at different rotational Reynolds numbers, it can be observed that an increase in the gap ratio results in a decrease in the tangential shear stress distribution on the rotating disk. Equation (18) describes the profile of the torque coefficient, which is influenced by the rotational Reynolds number, gap ratio and throughflow Reynolds number. Figure 6 shows how the torque coefficient on a rotating disk varies with varying rotational Reynolds numbers for three different throughflow Reynolds numbers, while keeping the gap ratio fixed at 0.0125, while Figure 7 illustrates the variation of the torque coefficient on a rotating disk at different rotational Reynolds numbers for three gap ratios, with a fixed throughflow Reynolds of 50.

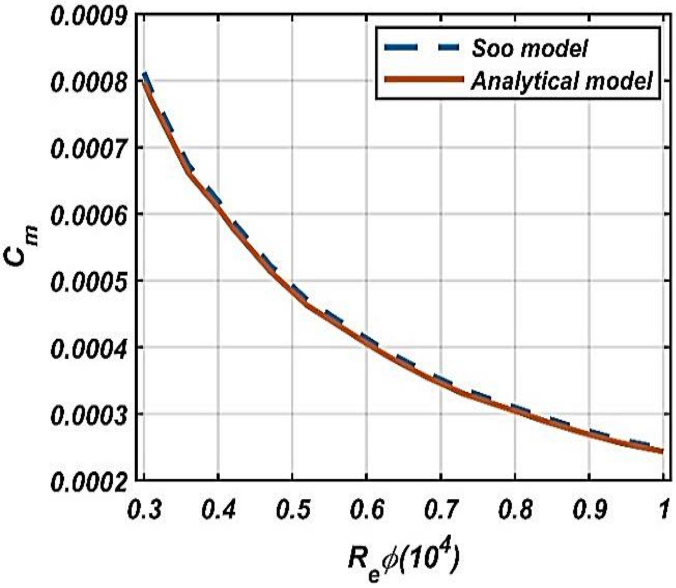

Figure 3. Comparison of analytical model with the Soo model.

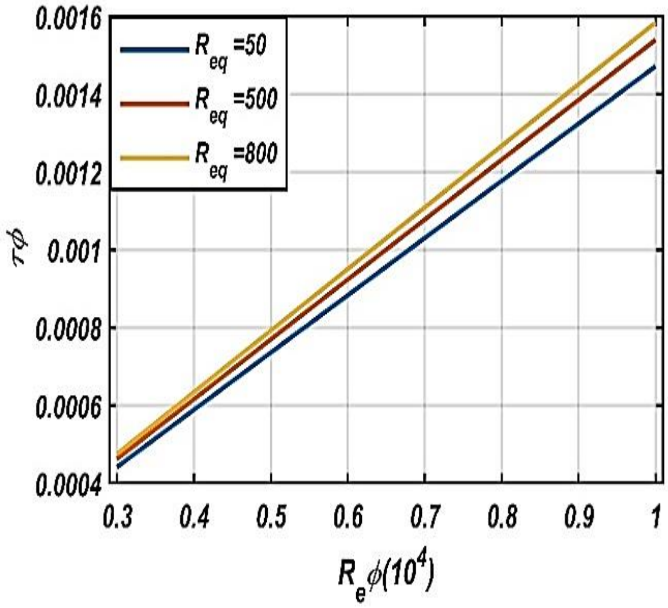

Figure 4. Variations of tangential Shear Stress with throughflow Reynolds number at fixed gap ratio 0.0125.

The trend of the torque coefficient is like shear stress distribution, which was also observe by other researchers. The variation of the torque coefficient with the rotational Reynolds number reveals a difference, where an increase in the rotational Reynolds number at a constant throughflow Reynolds number and gap ratio leads to a reduction in the torque coefficient. When the throughflow Reynolds number increases, the torque coefficient also increases. However, if the gap ratio increased from 0.0125 to 0.05, the torque coefficient decreases. Furthermore, the value of the torque coefficient drops more rapidly with an increase in the rotational Reynolds number.

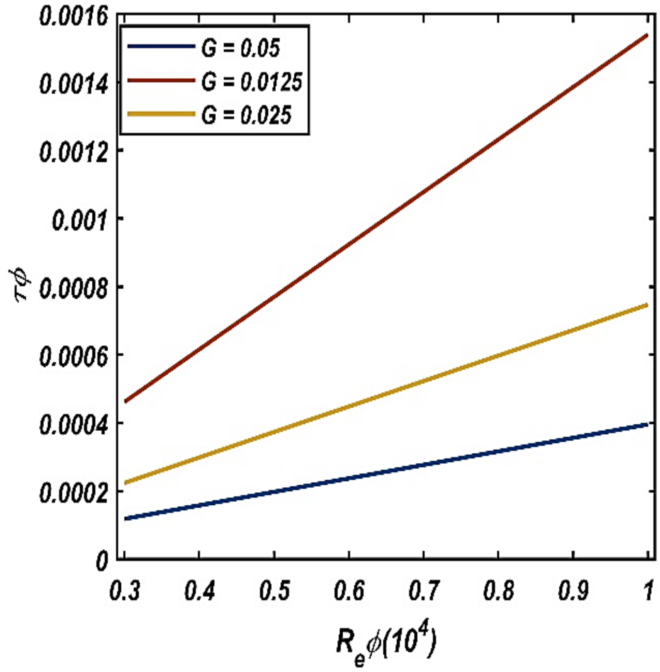

Figure 5. variation of tangential shear stress distribution with different gaps ratio at fixed throughflow Reynold number R_{eq} = 50.

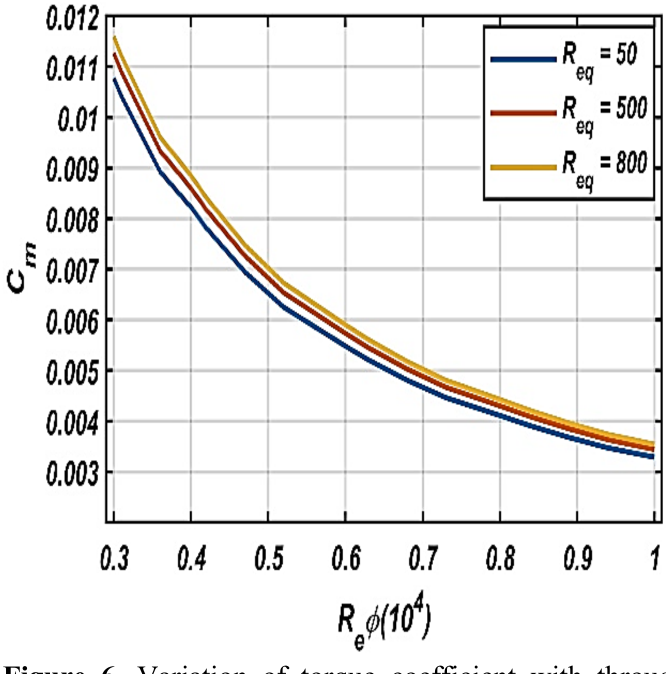

Figure 6. Variation of torque coefficient with throughflow Reynolds number at fixed gap ratio 0.0125.

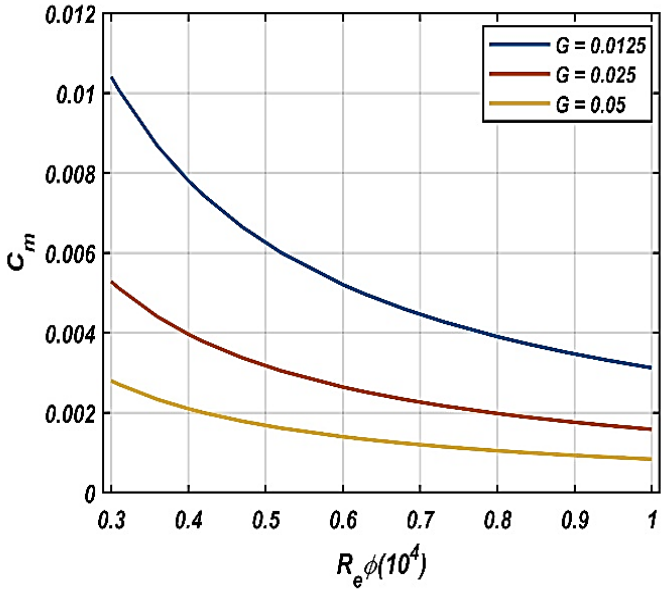

Figure 7. Variation of torque coefficient with different gaps ratio at fixed throughflow Reynold number R_{eq} = 50.

CONCLUSIONS

The available literature is consistent with the analytical model. An increase in throughflow Reynolds number leads to a corresponding increase in tangential shear stress, while the value of tangential shear stress also increases with rotational Reynolds number. The tangential shear stress value decreases at high gap ratios because of reduced viscous effects as compared to those at low gap ratios. Furthermore, as the throughflow Reynolds number rises, the torque coefficient also increases; however, an increase in the rotational Reynolds number leads to a decrease in the torque coefficient value. At low gap ratios, the value of torque coefficient drops more rapidly with a rise in rotational Reynolds number. The use of analytical model for shear stress and torque coefficient allows for the consideration of inward flow when designing smooth impellers for radial pumps and turbines.

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