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Analysis of Non-Uniform Damped Rayleigh Beam Resting on Pasternak Foundation Subjected to Distributed Load with Variable Axial Force

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Abstract

In this paper, damped non-uniform Rayleigh beam resting on Pasternak foundation and subjected to distributed load with variable axial force has been investigated. The solution techniques to the governing equation describing the dynamical system are based on Galerkin's method, Laplace integral transformation in conjunction with convolution theorem. Galerkin's method is explored to reduce the fourth order non-homogeneous partial differential equation to a second order ordinary differential equation. The resulting equation is then solved using Laplace transformation while the inverse Laplace transformed is obtained with the application of convolution theorem. The transverse displacement is calculated for various values of the axial force (N_o), Shear modulus (F_o), Foundation Stiffness (K_o), Damping Coefficient (Δc), and Rotatory Inertia (R_o). The results are shown graphically, and it reveals that the response amplitude of the beam under distributed load reduces with increase in axial force, shear modulus, foundation modulus, damping coefficient, and rotatory inertia. Also, as the length of the beam reduces the response amplitude of the beam decreases and this implies that there is direct relationship between the beam's response amplitude and the length of the beam. Finally, axial force and rotatory inertia have more noticeable effects on the beam subjected to the distributed compared with other structural parameters.

Keywords: Pasternak foundation, variable axial force, damped beam, non-uniform beam, distributed load

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INTRODUCTION

The analysis of elastic structural members resting on elastic foundation played a significant role in structural mechanics such as roadways, rails, and design. This area of research has drawn the attention of many researchers [1–10]. Those researchers considered one parameter foundation model which is the simplest model to describe soil behavior. A parameter model has one important shortcoming which is discontinuous behavior of the displacement in the surface beyond the region of the load, and this contravenes what happened in the real-life situation. Because of the shortcoming of one parameter model, two parameters foundation (bi-parametric subgrade) model came into existence and several researchers that considered two parameters foundation model with or without damping coefficient are Famuagun [11], Khalih and Ahmed [12], Mustapha [13], Ajibola [14], Saurabh [15], Baran [16], Jimoh and Ajoge [17] to mention but a few.

The above-mentioned researchers considered models with constant axial force and axial force may vary with spatial coordinate. In view of this, Jimoh [18] considered variable axial force influence on elastic beam under moving Load. He concluded that the response amplitude of the beam reduces with increase in the values of the parameters. Afolabi and Peter [19] used Galerkin's method and integral transformation to investigate variable axial force influence on elastic beam subjected to moving concentrated load. They concluded that higher values of the axial force significantly reduce as the response amplitude of the beam and resonance risk reduced because of the combined effects of the structural parameters.

Based on the above reviewed of literatures, no researcher considered variable axial force, damping coefficient, or Pasternak foundation on the same model.

Thus, this study investigated the damped non-uniform Rayleigh beam resting on Pasternak foundation and subjected to distributed load with variable axial force.

PROBLEM FORMULATION

With reference to Figure 1 below, the differential equation describing the dynamical system can be written as,

$$E\frac{\partial^2}{\partial x^2}\left\{J(x)\frac{\partial^2 W(x,t)}{\partial x^2}\right\}+M(x)\frac{\partial^2 W(x,t)}{\partial t^2}-\frac{\partial}{\partial x}\left\{N(x)\frac{\partial W(x,t)}{\partial x}\right\}-R_0^2\frac{\partial}{\partial x}\left\{M(x)\frac{\partial^3 W(x,t)}{\partial x\partial t^2}\right\}$$
$$+\Delta c\frac{\partial W(x,t)}{\partial t}+F_G(x,t)=P(x,t) \tag{1}$$

where, t is time coordinate, x is spatial coordinate, $P(x,t)$ is the transverse distributed load, Δc is damping coefficient, $N(x)$ is the variable axial force, $F_G(x,t)$ is the foundation reaction, R_o is the rotatory inertia, E is the young modulus of elasticity of the beam, $M(x)$ is variable mass per unit length of the beam, $J(x)$ is the variable moment of inertia of the cross section of the beam, and $W(x,t)$ is the response amplitude.

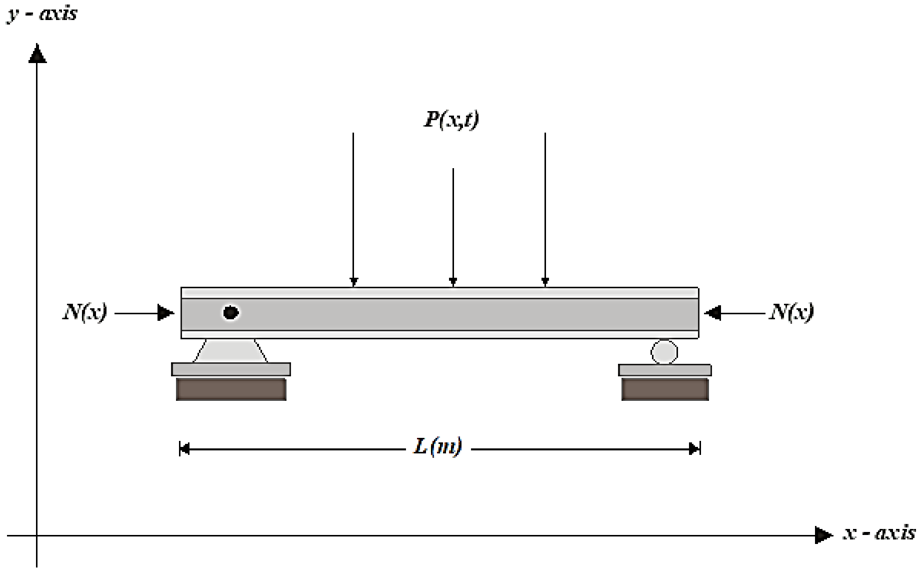

Figure 1. A non-uniform beam resting on Pasternak foundation with variable axial force.

This study considered simply supported boundary conditions given as,

$$W(0,t)=0=W(L,t)\ and\ \frac{\partial^2 W(0,t)}{\partial x^2}=0=\frac{\partial^2 W(L,t)}{\partial x^2} \tag{2}$$

and the initial conditions,

$$W(x,0) = 0 = \frac{\partial W(x,0)}{\partial t} \tag{3}$$

In Equation (1) above, distributed load of the form was considered,

$$P(x,t) = MgH(x-ct) \tag{4}$$

where, M is the mass of the distributed loads, g is the acceleration due to gravity, c is the constant velocity, t is time, and $H(x-ct)$ is the Heaviside function.

The Heaviside function $H(x-ct)$ is defined as,

$$H(x-ct) = \left\{\begin{smallmatrix} 0\,, x<ct \\ 1\,, x \geq ct \end{smallmatrix}\right. (ct \geq 0) \tag{5}$$

with properties:

i. $\frac{d}{dx}\{H(x-ct)\} = \delta(x-ct)$ (6)

ii. $H(x-ct)f(ct) = \begin{cases} 0\,, x < ct \\ f(ct)\,, x \geq ct \end{cases}$ (7)

The Pasternak foundation $F_G(x,t)$ is given by,

$$F_G(x,t) = K_o W(x,t) - F_o \frac{\partial^2 W(x,t)}{\partial x^2} \tag{8}$$

where, K_o and F_o are constants foundation stiffness and shear modulus, respectively.

The variable moment of inertia $J(x)$ and variable mass per unit length of the beam $M(x)$ are, respectively, given as,

$$J(x) = J_o\left(1 + sin\frac{\pi x}{L}\right)^3 \tag{9}$$

$$M(x) = M_o\left(1 + sin\frac{\pi x}{L}\right) \tag{10}$$

where, J_0 and M_0 are constant moment of inertia and constant mass of the beam, respectively.

The variable axial force $N(x)$ is given as,

$$N(x) = N_o\left(1 + sin\frac{\pi x}{L}\right) \tag{11}$$

where, N_o is the constant axial force.

Substituting Equations (4), (8), (9), (10), and (11) into Equation (1) to obtain,

$$E\frac{\partial^2}{\partial x^2}\left\{I_o\left(1 + sin\frac{\pi x}{L}\right)^3 \frac{\partial^2 Y(x,t)}{\partial x^2}\right\} + \mu_o\left(1 + sin\frac{\pi x}{L}\right)\frac{\partial^2 Y(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left\{N_o\left(1 + sin\frac{\pi x}{L}\right)\frac{\partial Y(x,t)}{\partial x}\right\} - R_0^2\frac{\partial}{\partial x}\left\{\mu_o(1 + sin\frac{\pi x}{L})\frac{\partial^3 Y(x,t)}{\partial x \partial t^2}\right\} + \Delta c\frac{\partial Y(x,t)}{\partial t} - F_o\frac{\partial^2 Y(x,t)}{\partial x^2} + K_o Y(x,t) = MgH(x-ct) \tag{12}$$

Further simplification of Equation (12) to obtain,

$$EI_o\left(\frac{5}{2}+\frac{15}{4}sin\frac{\pi x}{L}-\frac{1}{4}sin\frac{3\pi x}{L}-\frac{3}{2}cos\frac{2\pi x}{L}\right)\frac{\partial^4 Y(x,t)}{\partial x^4}+ \\ EI_o\left(\frac{9\pi^2}{4L^2}sin\frac{3\pi x}{L}-\frac{15\pi^2}{4L^2}sin\frac{\pi x}{L}+\frac{6\pi^2}{L^2}cos\frac{2\pi x}{L}\right)\frac{\partial^2 Y(x,t)}{\partial x^2}+\mu_o\frac{\partial^2 Y(x,t)}{\partial t^2}+\mu_o sin\frac{\pi x}{L}\frac{\partial^2 Y(x,t)}{\partial t^2}-N_o\frac{\partial^2 Y(x,t)}{\partial x^2}- \\ N_o sin\frac{\pi x}{L}\frac{\partial^2 Y(x,t)}{\partial x^2}-N_o\frac{\pi}{L}cos\frac{\pi x}{L}\frac{\partial Y(x,t)}{\partial x}-\mu_o {R_o}^2\frac{\partial^4 Y(x,t)}{\partial x^2\partial t^2}-\mu_o {R_o}^2 sin\frac{\pi x}{L}\frac{\partial^4 Y(x,t)}{\partial x^2\partial t^2}- \\ \mu_o {R_o}^2\frac{\pi}{L}cos\frac{\pi x}{L}\frac{\partial^3 Y(x,t)}{\partial x\partial t^2}+\Delta c\frac{\partial Y(x,t)}{\partial t}-F_o\frac{\partial^2 Y(x,t)}{\partial x^2}+K_o Y(x,t)\ =\ MgH(x-ct) \tag{13}$$

Equations (13) is the governing equation describing the dynamical system.

METHOD OF SOLUTION

The best method for solving diverse problems involving mechanical vibration is Galerkin's method. The method requires a solution of the form,

$$Y_j(x,t)=\sum_{j=1}^{\infty}q_j(t)\beta_j(x) \tag{14}$$

where $q_j(t)$'s are the coordinates in modal space and $\beta_j(x)$'s are the normal modes of free vibration of the beam which can be written as,

$$\beta_j(x)=sin\theta_j x+A_j cos\theta_j x+B_j sinh\theta_j x+C_j cosh\theta_j \tag{15}$$

where, the constants A_j, B_j, C_j and the mode frequencies θ_j defines the space and amplitude of the beam vibration whose values depend on the associated boundary conditions.

Thus, for a beam with simple support at both ends, it can be shown that,

$$A_j=B_j=C_j=0 \text{ and } \theta_j=\frac{j\pi}{L} \tag{16}$$

and

$$\mathrm{\beta_j(x)=sin\frac{j\pi x}{L}} \tag{17}$$

By substituting Equation (14) into Equation (13) and simplified to obtain,

$$\sum_{j=1}^{n}\begin{bmatrix} EI_o\left(\frac{5}{2}+\frac{15}{4}sin\frac{\pi x}{L}-\frac{1}{4}sin\frac{3\pi x}{L}-\frac{3}{2}cos\frac{2\pi x}{L}\right)q_j(t)\ \beta_j^{iv}(x)+ \\ EI_o\left(\frac{9\pi^2}{4L^2}sin\frac{3\pi x}{L}-\frac{15\pi^2}{4L^2}sin\frac{\pi x}{L}+\frac{6\pi^2}{L^2}cos\frac{2\pi x}{L}\right)\ q_j(t)\ \beta_j^{11}(x)+ \\ \mu_o\ddot{q}(t)\beta(x)+\mu_o sin\frac{\pi x}{L}\ddot{q}(t)-N_o q_j(t)\ \beta_j^{11}(x)-N_o sin\frac{\pi x}{L}q_j(t)\ \beta_j^{11}(x) \\ -N_o\frac{\pi}{L}cos\frac{\pi x}{L}q_j(t)\ \beta_j^{11}(x)-\mu_o {R_o}^2\ddot{q}(t)\ \beta_j^{11}(x)-\mu_o {R_o}^2 sin\frac{\pi x}{L}\ddot{q}(t)\ \beta_j^{11}(x)- \\ -\mu_o {R_o}^2\frac{\pi}{L}cos\frac{\pi x}{L}\ddot{q}(t)\beta_j^{1}(x)-+\Delta c\dot{q}_j(t)\beta(x)-F_o\ q_j(t)\ \beta_j^{11}(x)+K_o q_j(t)\beta(x)\end{bmatrix}$$

$$-MgH(x-ct)=0 \tag{18}$$

To obtain the coordinates $q_j(t)$ in modal space, the Galerkin's method requires that Equation (18) be orthogonal to the function $\beta_k(t) = sin\frac{k\pi x}{L}$ where k is a dummy index.

By applying the orthogonality conditions and neglecting the sigma sign and substituting Equation (17) and its derivative into Equation (18) to obtain,

$$\left[\mu_o\left(1+{R_o}^2\left(\frac{j\pi}{L}\right)^2\right)I_1+\mu_o\left(1+{R_o}^2\mathrm{j}\left(\frac{\pi}{L}\right)^2\right)I_2-\mu_o{R_o}^2\left(\frac{j\pi}{L}\right)^2 I_3\right]\ddot{q}_j(t)+\Delta cI_1\dot{q}_j(t)$$

$$\begin{bmatrix} EI_o\left(\frac{j\pi}{L}\right)^4\left(\frac{5}{2}I_1+\frac{15}{4}I_2-\frac{1}{4}I_4-\frac{3}{2}I_5\right)-EI_o\left(\frac{j\pi}{L}\right)^2\left(\frac{9\pi^2}{4L^2}I_4+\frac{15\pi^2}{4L^2}I_2+\frac{6\pi^2}{L^2}I_5\right) \\ +N_o\left(\frac{j\pi}{L}\right)^2 I_1+N_o\left(\frac{j\pi}{L}\right)^2 I_2-N_o\mathrm{j}\left(\frac{\pi}{L}\right)^2 I_3-k_oI_1 \\ \end{bmatrix} q_j(t)=\mathrm{Mg}I_6 \tag{19}$$

where

$$\left.\begin{aligned} I_1 &= \int_0^L sin\frac{j\pi x}{L}sin\frac{k\pi x}{L}dx\ ,\ I_2=\int_0^L sin\frac{\pi x}{L}sin\frac{j\pi x}{L}sin\frac{k\pi x}{L}dx \\ I_3 &= \int_0^L cos\frac{\pi x}{L}cos\frac{j\pi x}{L}sin\frac{k\pi x}{L}dx,\ I_4=\int_0^L sin\frac{3\pi x}{L}sin\frac{j\pi x}{L}sin\frac{k\pi x}{L}dx \\ I_5 &= \int_0^L cos\frac{2\pi x}{L}sin\frac{j\pi x}{L}sin\frac{k\pi x}{L}dx,\ I_6=\int_0^L H(x-ct)Sin\frac{k\pi x}{L}\ dx \end{aligned}\right\} \tag{20}$$

By evaluating the integrals ($I_1 - I_6$) for $j = k$ to obtain,

$$I_1 = \frac{\mathrm{L}}{2} \tag{21a}$$

$$I_2 = \frac{\mathrm{L}}{4\pi}\left(2(1-cos\pi)+\frac{cos(1+2\mathrm{j})\pi-1}{(1+2\mathrm{j})}+\frac{cos(1-2\mathrm{j})\pi-1}{(1-2\mathrm{j})}\right) \tag{21b}$$

$$I_3 = \frac{\mathrm{L}}{4\pi}\left(\frac{1-cos(1+2\mathrm{j})\pi}{1+2\mathrm{j}}+\frac{cos(1-2\mathrm{j})\pi-1}{1-2\mathrm{j}}\right) \tag{21c}$$

$$I_4 = \frac{\mathrm{L}}{4\pi}\left(\frac{cos(3+2\mathrm{j})\pi-1}{3+2j}+\frac{cos(3-2\mathrm{j})\pi-1}{3-2j}-\frac{2}{3}(cos3\pi-1)\right) \tag{21d}$$

$$I_5 = \frac{\mathrm{L}}{8\pi}\left(2sin2\pi-\frac{sin(1+\mathrm{j})2\pi}{1+j}-\frac{\sin(1-j)2\pi}{1-j}\right) \tag{21e}$$

$$I_6 = \frac{L}{k\pi}\left(cos\frac{k\pi ct}{L}-cosk\pi\right) \tag{21f}$$

By substituting the results of the integrals in (21a–21f) into Equation (19) to obtain,

$$\ddot{q}_j(t)+\beta_{22}\dot{q}_j(t)+\beta_{33}q_j(t)=\beta_{44}(cos\beta_ot+1) \tag{22}$$

where

$$\left.\begin{aligned} & d_1 = \mu_o {R_o}^2 \left(\frac{j\pi}{L}\right)^2, d_2 = \mu_o {R_o}^2 j \left(\frac{\pi}{L}\right)^2, d_3 = EI_o \left(\frac{j\pi}{L}\right)^4 \\ & d_4 = EI_o \left(\frac{j\pi}{L}\right)^2, d_5 = N_o \left(\frac{j\pi}{L}\right)^2, d_6 = N_o j \left(\frac{j\pi}{L}\right)^2, d_7 = F_o \left(\frac{j\pi}{L}\right)^2 \\ & \beta_{11} = (\mu_o + d_1)(I_1 + I_2) - d_2 I_3 \end{aligned}\right\} \quad (23), (24)$$

$$\beta_{12} = \Delta c I_1 \tag{25}$$

$$\beta_{13} = d_3 \left(\frac{5}{2} I_1 + \frac{15}{4} I_2 - \frac{1}{4} I_4 - \frac{3}{2} I_5\right) - d_4 \left(\frac{9\pi^2}{4L^2} I_4 - \frac{15\pi^2}{4L^2} I_2 + \frac{6\pi^2}{L^2} I_5\right) + (d_5 + d_7 + K_o) I_1 + d_5 I_2 - d_6 I_3 \tag{26}$$

$$\beta_{22} = \frac{\beta_{12}}{\beta_{11}}, \beta_{33} = \frac{\beta_{13}}{\beta_{11}}, \beta_{44} = \frac{Mg}{\beta_{11}} \text{ and } \beta_o = \frac{k\pi ct}{L} \tag{27}$$

Equation (22) above is a differential equation of second order and subjecting it to the Laplace transformation defined by,

$$\mathcal{L}\{q_j(t)\} = Q_j(s) = \int_0^\infty e^{-st} q_j(t) dt \tag{28}$$

With the initial conditions in Equation (3), Equation (22) becomes,

$$s^2 Q_j(s) + \beta_{22} s Q_j(s) + \beta_{33} Q_j(s) = \beta_{44} \left(\frac{s}{s^2 + {\beta_o}^2} + \frac{1}{s}\right) \tag{29}$$

After simplification and rearrangement, Equation (29) can be expressed as,

$$Q_j(s) = \frac{\beta_{44}}{r_1 - r_2} \left\{\left(\frac{1}{s - r_1}\right)\left(\frac{s}{s^2 + {\beta_o}^2}\right) - \left(\frac{1}{s - r_2}\right)\left(\frac{s}{s^2 + {\beta_o}^2}\right)\right\} + \frac{\beta_{44}}{r_1 - r_2} \left\{\left(\frac{1}{s - r_1}\right)\left(\frac{1}{s}\right) - \left(\frac{1}{s - r_2}\right)\left(\frac{1}{s}\right)\right\} \tag{30}$$

where

$$r_1 = -\frac{\beta_{22}}{2} + \frac{\sqrt{{\beta_{22}}^2 - 4\beta_{33}}}{2}, r_2 = -\frac{\beta_{22}}{2} - \frac{\sqrt{{\beta_{22}}^2 - 4\beta_{33}}}{2} \tag{31}$$

Laplace inverse of Equation (30) can be obtained by using the following,

$$M_1(s) = \frac{1}{s - r_1}, M_2(s) = \frac{1}{s - r_2} \tag{32}$$

$$N_1(s) = \frac{s}{s^2 + {\beta_o}^2}, N_2(s) = \frac{1}{s} \tag{33}$$

Substituting Equations (32) and (33) into Equation (30) to obtain,

$$Q_j(s) = \frac{\beta_{44}}{r_1 - r_2}\{M_1(s)N_1(s) - M_2(s)N_1(s)\} + \frac{\beta_{44}}{r_1 - r_2}\{M_1(s)N_2(s) - M_2(s)N_2(s)\} \tag{34}$$

The convolution of $M_i's$ and $N_i's$ which is the inversion of (34) can be defined as,

$$(M_i * N_j)(t) = \int_0^t M_i(t-v)N_j(v)dv\,, i = 1,2. j = 1,2.] \tag{35}$$

where

$$M_1(t-v) = e^{r_1(t-v)}\,,\ M_2(t-v) = e^{r_2(t-v)} \tag{36}$$

$$N_1(v) = cos\beta_o v\,,\ N_2(v) = 1 \tag{37}$$

Substituting Equations (36) and (37) into Equation (34) and applying the convolution theorem in Equation (35) and after simplifying to obtain,

$$\begin{aligned} q_j(t) &= TT_1(e^{r_1 t} - cos\beta_o t) - TT_2(e^{r_2 t} - cos\beta_o t) + T(T_3 - T_4)sin\beta_o t \\ &+TT_5(e^{r_1 t} - 1 - TT_6(e^{r_2 t} - 1) \end{aligned} \tag{38}$$

where

$$\left.\begin{aligned} T = \frac{\beta_{44}}{r_1 - r_2}, T_1 = \frac{r_1}{\beta_o{}^2 + r_1{}^2}, T_2 = \frac{r_2}{\beta_o{}^2 + r_2{}^2}, \\ T_3 = \frac{\beta_o}{\beta_o{}^2 + r_1{}^2}, T_4 = \frac{\beta_o}{\beta_o{}^2 + r_2{}^2}, T_5 = \frac{1}{r_1}, T_6 = \frac{1}{r_2} \end{aligned}\right\} \tag{39}$$

Substituting Equations (17) and (38) into Equation (14) to obtain,

$$\begin{aligned} Y_j(x,t) = \sum_{j=1}^{\infty} &(TT_1(e^{r_1 t} - cos\beta_o t) - TT_2(e^{r_2 t} - cos\beta_o t) + T(T_3 - T_4)sin\beta_o t + TT_5(e^{r_1 t} - 1) - TT_6(e^{r_2 t} \\ &- 1))sin\frac{j\pi x}{L} \end{aligned} \tag{40}$$

Equation (40) is the response amplitude of the beam on bi-parametric subgrade with variable axial force and damping coefficient under distributed loads.

RESULTS AND DISCUSSION

To discuss the closed form solutions of the analysis carried out in this work, the following values are used:

$$L = 12.9m, M = 8407.28\ \mathrm{kgm}^{-1}, c = 8.128\ \mathrm{ms}^{-1}, E = 2.109 * 10^9 Kgm^{-2},$$

$$I_o = 2.87698 * 10^{-3}\mathrm{m}^4, \mu_o = 4501.563 Kgm^{-1}.$$

Figure 2 shows the transverse displacement of the beam and it reduces as the axial force (N_o) increases. Figure 3 also shows that increases in foundation stiffness (K_o) will lead decrease in the response amplitude of the beam. Similarly, Figures 4, 5, and 6 show that in each case, an increase in the shear modulus (F_o), damping coefficient (Δc), and rotatory inertia (R_o) decrease the transverse displacement of the beam subjected to distributed moving load. Finally, the axial force and rotatory inertia have higher influences on the beam compared to other parameters.

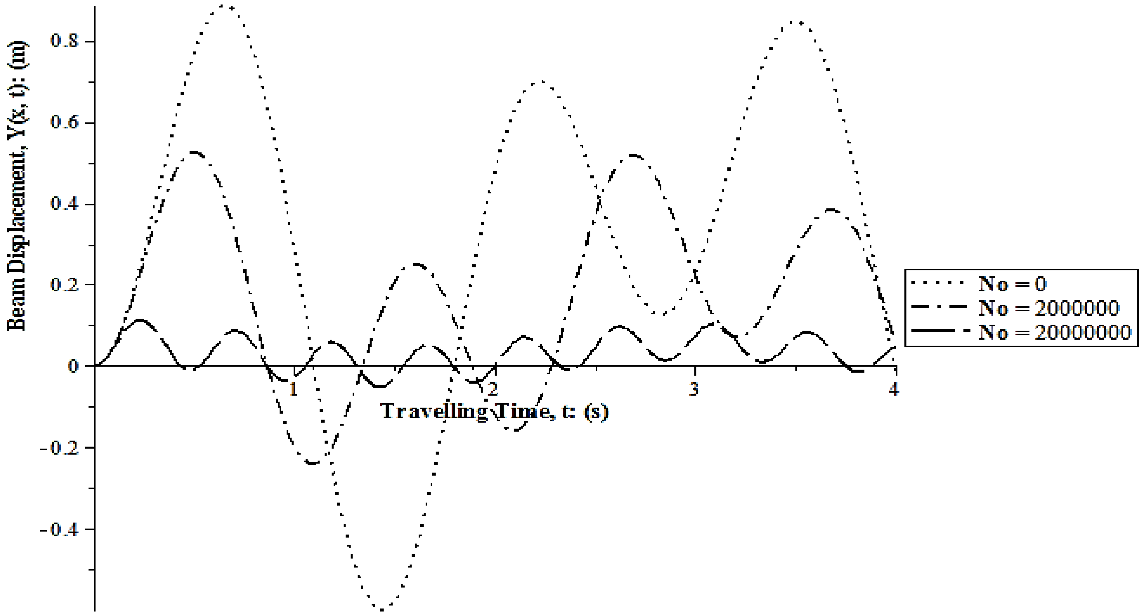

Figure 2. Response amplitude of the beam for different values of the axial force (N_o).

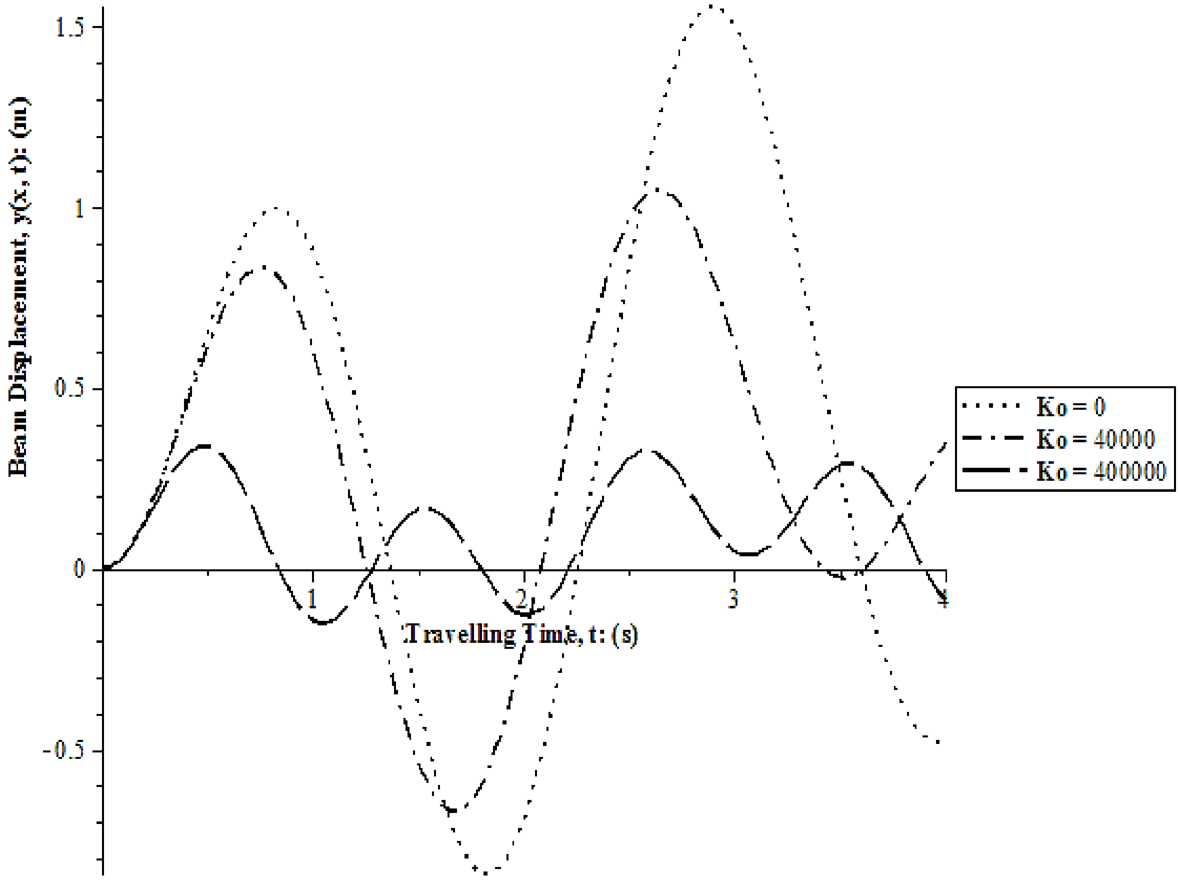

Figure 3. Response amplitude of the beam for different values of the foundation stiffness (K_o).

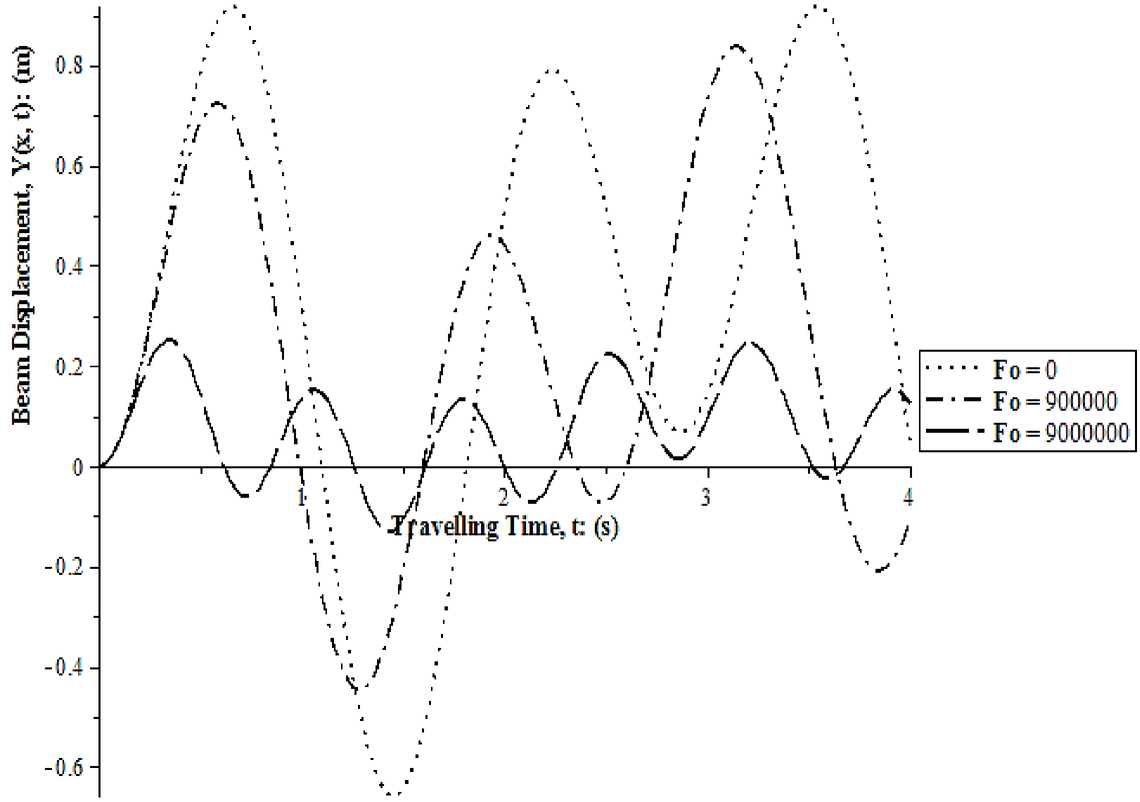

Figure 4. Response amplitude of the beam for different values of the shear modulus (F_o).

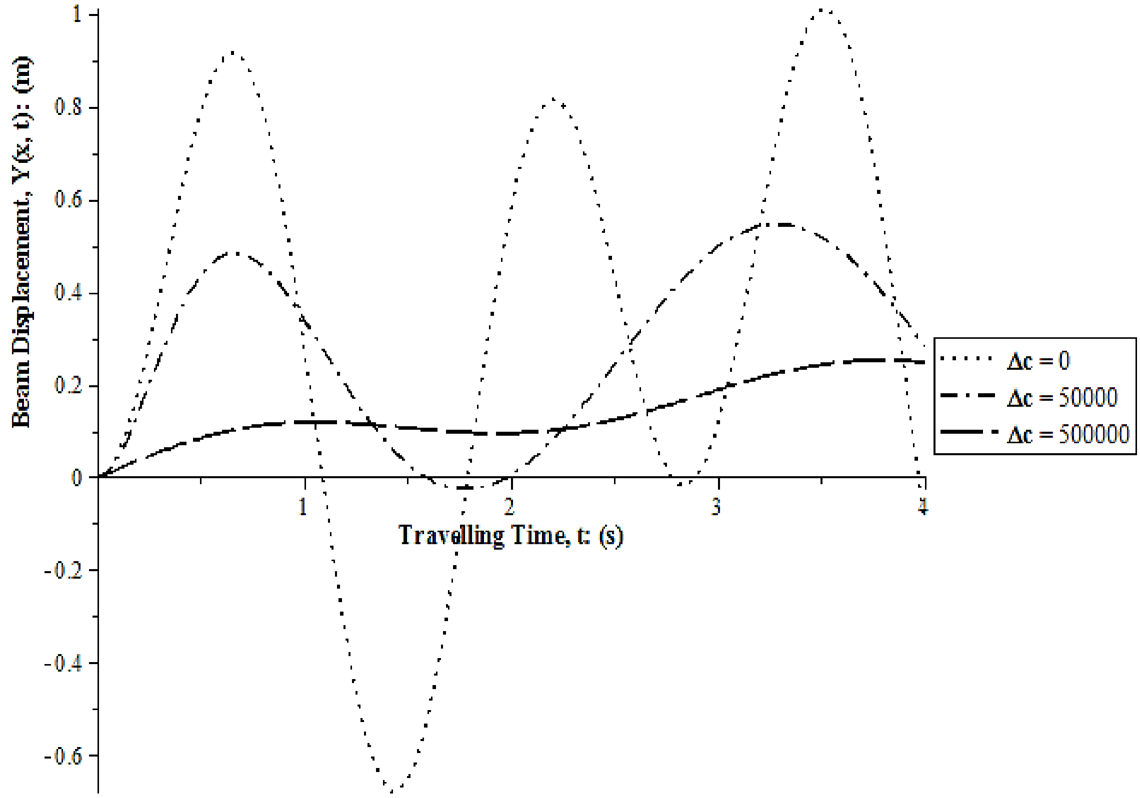

Figure 5. Response amplitude of the beam for different values of the damping coefficient (Δc).

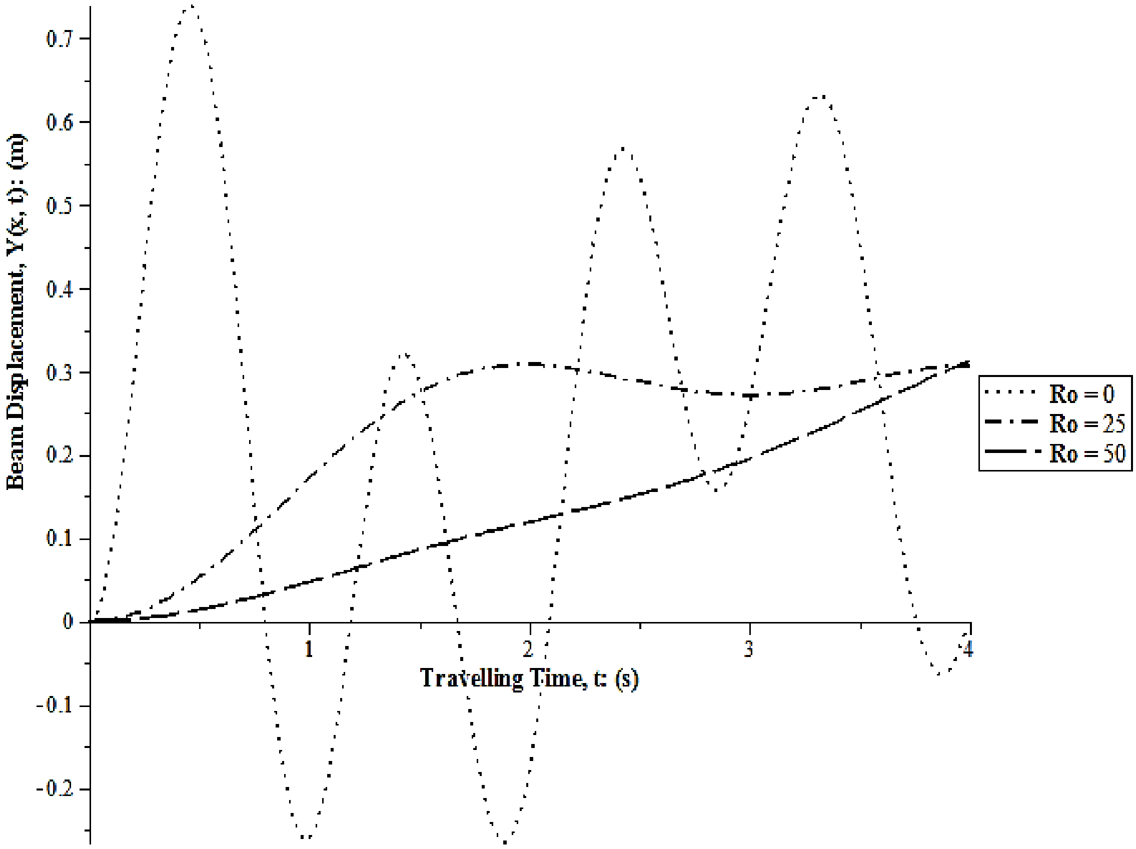

Figure 6. Response amplitude of the beam for different values of the rotatory inertia (R_o).

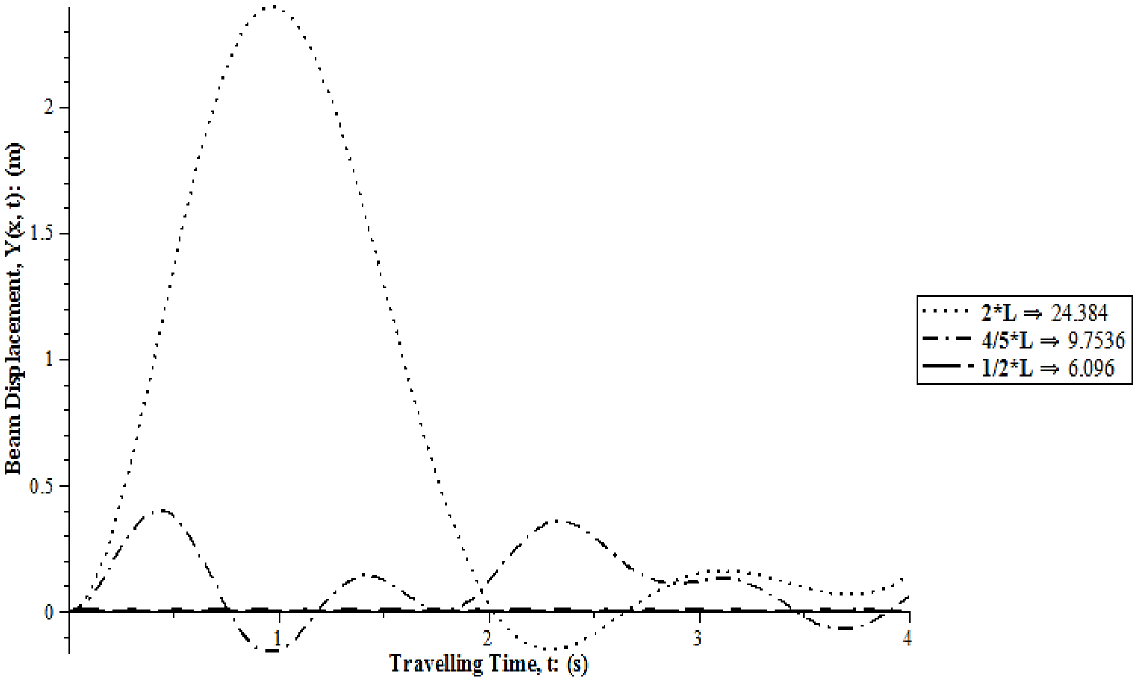

Figure 7. Response amplitude of the beam for different values of the length of the beam (L).

CONCLUSIONS

The dynamics response of damped non-uniform elastic beam on Pasternak foundations subjected to distributed load with variable axial force has been investigated. Galerkin's method, Laplace integral in conjunction with convolution theorem are used to obtain the solutions in plotted form. The results as presented in the plotted graphs reveals that increases in the values of axial force, foundation stiffness, shear modulus, damping coefficient, and rotatory inertia reduces the response amplitudes of the beam. Finally, length of the beam and deflection profile of the beam are directly related, and this is shown in Figure 7 above. Also, high risk of resonance effect could be reduced because of the high values of the structural parameters.

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